

Evaluate each function

$$h(n) = -2n^2 + 4; \text{ Find } h(4)$$

$$\begin{aligned} h(4) &= -2(4)^2 + 4 \\ &= -28 \end{aligned}$$

$$p(a) = -4^{3a}; \text{ Find } p(-1)$$

$$\begin{aligned} p(-1) &= -4^{3(-1)} \\ &= -4^{-3} \\ &= \frac{1}{-4^3} = -\frac{1}{64} \end{aligned}$$

$$f(x) = x^2 - 3x; \text{ Find } f(-8)$$

$$\begin{aligned} f(-8) &= (-8)^2 - 3(-8) \\ &= 64 + 24 \\ &= 88 \end{aligned}$$

$$p(a) = a^3 - 5; \text{ Find } p(x-4)$$

$$\begin{aligned} p(x-4) &= (x-4)^3 - 5 \\ &= (x-4)(x-4)(x-4) \\ &= (x-4)(x^2 - 8x + 16) \\ &= x^3 - 8x^2 + 16x \\ &\quad - 4x^2 + 32x - 64 \\ &= x^3 - 12x^2 + 48x - 64 - 5 \\ &= x^3 - 12x^2 + 48x - 69 \end{aligned}$$

## COMPOSITION OF FUNCTIONS

The composition of functions  $f$  and  $g$  is written  $f \circ g$  and is defined by

$$(f \circ g)(x) = f(g(x))$$

We read  $f(g(x))$  as  $f$  of  $g$  of  $x$ .

**EXAMPLE 10.2**

For functions  $f(x) = x^2 - 4$ , and  $g(x) = 3x + 2$ , find: (a)  $(f \circ g)(-3)$ , (b)  $(g \circ f)(-1)$ , and (c)  $(f \circ f)(2)$ .

$$\begin{aligned} \text{a) } (f \circ g)(-3) &= f(g(-3)) \\ (f \circ g)(-3) &= 45 \end{aligned}$$

$\begin{aligned} \hookrightarrow g(-3) &= 3(-3) + 2 \\ &= -7 \end{aligned}$

$$\begin{aligned} f(-7) &= (-7)^2 - 4 \\ &= 45 \end{aligned}$$

$$\begin{aligned} \text{b) } (g \circ f)(-1) \\ (g \circ f)(-1) &= -7 \end{aligned}$$

$$\begin{aligned} f(-1) &= (-1)^2 - 4 \\ &= -3 \\ g(-3) &= 3(-3) + 2 \\ &= -7 \end{aligned}$$

$$\begin{aligned} \text{c) } (f \circ f)(2) \\ (f \circ f)(2) &= -4 \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^2 - 4 \\ &= 0 \\ f(0) &= 0^2 - 4 \\ &= -4 \end{aligned}$$

For functions  $f(x) = x^2 + 1$ , and  $g(x) = 3x - 5$ , find a)  $(f \circ g)(-1)$ , b)  $(g \circ f)(2)$ , and c)  $(f \circ f)(-1)$ .

$$\begin{array}{lll} \text{a) } (f \circ g)(-1) = f(g(-1)) & g(-1) = 3(-1) - 5 & f(-8) = (-8)^2 + 1 \\ & = -8 & = \boxed{65} \end{array}$$

$$\begin{array}{lll} \text{b) } (g \circ f)(2) = g(f(2)) & f(2) = (2)^2 + 1 & g(5) = 3(5) - 5 \\ & = 5 & = \boxed{10} \end{array}$$

$$\begin{array}{lll} (f \circ f)(-1) = f(f(-1)) = & f(-1) = (-1)^2 + 1 & f(2) = 2^2 + 1 \\ & = 2 & = \boxed{5} \end{array}$$

For functions  $f(x) = 4x - 5$  and  $g(x) = 2x + 3$ , find: a)  $(f \circ g)(x)$ , b)  $(g \circ f)(x)$ , and c)  $(f \cdot g)(x)$ .

$$\begin{aligned} \text{a) } (f \circ g)(x) &= f(g(x)) = 4x - 5 \\ &= f(2x + 3) = 4(2x + 3) - 5 \\ &= 8x + 12 - 5 \\ &= \boxed{8x + 7} \end{aligned}$$

$$\begin{aligned} \text{b) } (g \circ f)(x) &= g(f(x)) = 2x + 3 \\ &= g(4x - 5) = 2(4x - 5) + 3 \\ &= 8x - 10 + 3 \\ &= \boxed{8x - 7} \end{aligned}$$

$$\begin{aligned} \text{c) } (f \cdot g)(x) &= (4x - 5)(2x + 3) \\ &= 8x^2 + 12x - 10x - 15 \\ &= 8x^2 + 2x - 15 \end{aligned}$$

For functions  $f(x) = 4x - 3$ , and  $g(x) = 6x - 5$ , find a)  $(f \circ g)(x)$ , b)  $(g \circ f)(x)$ , and c)  $(f \cdot g)(x)$ .

$$a) (f \circ g)(x) = f(g(x)) = 4x - 3$$

$$\begin{aligned} f(6x-5) &= 4(6x-5) - 3 \\ &= 24x - 20 - 3 \\ &= 24x - 23 \end{aligned}$$

$$b) (g \circ f)(x) = g(f(x)) = 6x - 5$$

$$\begin{aligned} g(4x-3) &= 6(4x-3) - 5 \\ &= 24x - 18 - 5 \\ &= 24x - 23 \end{aligned}$$

$$c) (f \cdot g)(x) = f(x) \cdot g(x)$$

$$(4x-3)(6x-5)$$

$$24x^2 - 20x - 18x + 15$$

$$24x^2 - 38x + 15$$

1, 3, 5, 9, 11

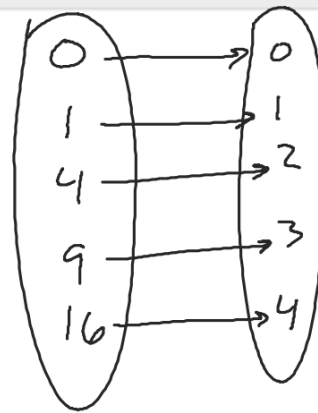
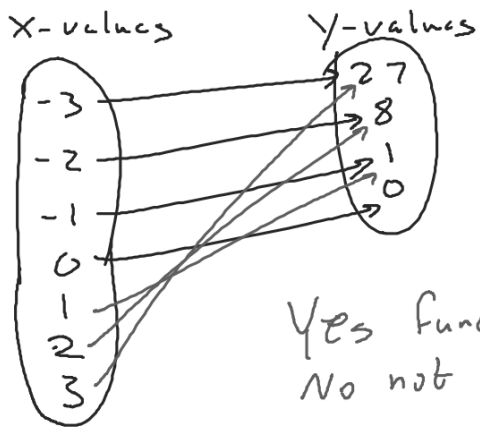
## ONE-TO-ONE FUNCTION

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A function is **one-to-one** if each value in the range corresponds to one element in the domain. For each ordered pair in the function, each  $y$ -value is matched with only one  $x$ -value. There are no repeated  $y$ -values.

For each set of ordered pairs, determine if it represents a function and, if so, if the function is one-to-one.

Ⓐ  $\{(-3, 27), (-2, 8), (-1, 1), (0, 0), (1, 1), (2, 8), (3, 27)\}$  and Ⓑ  $\{(0, 0), (1, 1), (4, 2), (9, 3), (16, 4)\}$ .

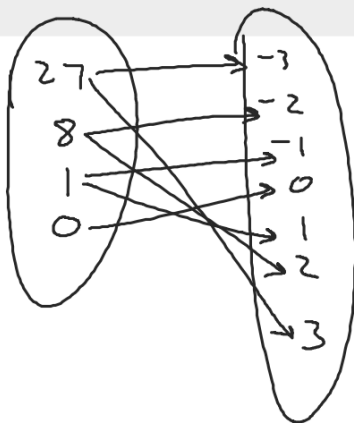




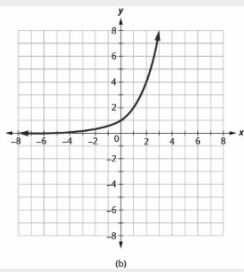
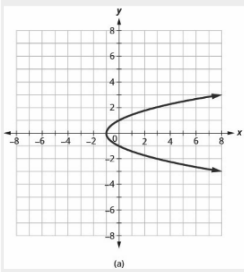
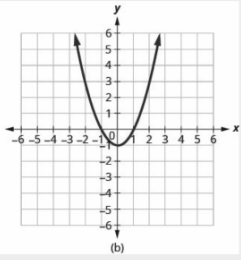
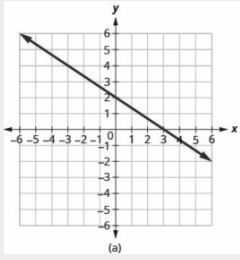
For each set of ordered pairs, determine if it represents a function and if so, is the function one-to-one.

Ⓐ  $\{(27, -3), (8, -2), (1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$  Ⓑ

$\{(7, -3), (-5, -4), (8, 0), (0, 0), (-6, 4), (-2, 2), (-1, 3)\}$



Determine Ⓐ whether each graph is the graph of a function and, if so, Ⓟ whether it is one-to-one.



13, 17-20